1- Risolvi le seguenti equazioni intere e scrivi le soluzioni reali in ordine crescente. Indica anche se ci sono soluzioni non reali e quante sono:

$17x = (3x+1)(6-x)$ $\qquad$ $17x(3x-1)^2 = 0$ $\qquad$ $7x^3 = 8x - 1$ $\qquad$ $4x^5 + 4x^3 = 0$

- $17x = (3x+1)(6-x)$

$17x = 18x - 3x^2 + 6 - x$

$17x - 18x + 3x^2 - 6 + x = 0$

$3x^2 - 6 = 0$

$3(x^2 - 2) = 0$

$S = \{-\sqrt{2}; \sqrt{2}\}$

$\Delta = b^2 - 4ac = -4(1)(-2) = 8$

$x_{1,2} = \dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{\pm\sqrt{8}}{2} \left< \begin{array}{l} \frac{+\sqrt{8}}{2} \\ \frac{-\sqrt{8}}{2} \end{array} \right.$

$\dfrac{+\sqrt{8}}{2} = \dfrac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$

$\dfrac{-\sqrt{8}}{2} = \dfrac{-2\sqrt{2}}{2} = -\sqrt{2}$

- $17x(3x-1)^2 = 0$

$x = 0$

$\dfrac{\cancel{3}x}{\cancel{3}} = \dfrac{1}{3} \Rightarrow \quad x = \dfrac{1}{3}$ doppia

$S = \left\{0; \dfrac{1}{3} \text{ (doppia)}\right\}$

- $7x^3 = 8x - 1$

$7x^3 - 8x + 1 = 0$

$(x-1)(7x^2 + 7x - 1) = 0$

$S = \left\{\dfrac{-7-\sqrt{77}}{14}; \dfrac{-7+\sqrt{77}}{14}; 1\right\}$

$P(-1) = 7(-1)^3 - 8(-1) + 1 = -7 + 8 + 1$

$P(1) = 7 - 8 + 1 = 0$

	x^3	x^2	x	tn
	7	0	−8	1
1		7	7	−1
	7	7	−1	0
	x^2	x	tn	

$\Delta = b^2 - 4ac = 49 - 4(-7) = 49 + 28 = 77$

$x_{1,2} = \dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{-7 \pm \sqrt{77}}{14} = \left< \begin{array}{l} \frac{-7+\sqrt{77}}{14} \\ \frac{-7-\sqrt{77}}{14} \end{array} \right.$

- $4x^5 + 4x^3 = 0$

$4x^3(4x^2 + 1) = 0$

$S = \{0 \text{ (tripla)}\}$ e due sol $\notin \mathbb{R}$

$\Delta = b^2 - 4ac = -4(1) = -4$

2- Risolvi le seguente equazioni fratte: (/25 punti)

$$\frac{x^2-x}{x^2+6x+9}+\frac{3}{3x+9}=0 \qquad \frac{3(x^2-1)}{3x^2-5x+2}+\frac{3x-1}{2-3x}+2=0$$

- $\dfrac{x^2-x}{x^2+6x+9}+\dfrac{3}{3x+9}=0$

$\dfrac{x^2-x}{(x+3)^2}+\dfrac{\cancel{3}}{\cancel{3}(x+3)}=0$

D $\dfrac{x^2\cancel{-x}\cancel{+x}+3}{\cancel{(x+3)^2}}=0\cdot D$ C.E. $x\neq -3$

$x^2+3=0$

$\Delta=b^2-4ac=-4(1)(+3)=-12$

2 sol $\notin \mathbb{R}$

- $\dfrac{3(x^2-1)}{3x^2-5x+2}+\dfrac{3x-1}{2-3x}+2=0$

$\dfrac{3(x^2-1)}{3x(x-1)-2(x-1)}+\dfrac{3x-1}{2-3x}+2=0$

$\dfrac{3x^2-3}{(x-1)(3x-2)}+\dfrac{3x-1}{2-3x}+2=0$

$\dfrac{3x^2-3}{(x-1)(3x-2)}-\dfrac{3x-1}{3x-2}+2=0$ C.E. $x\neq 1 \wedge x\neq \frac{2}{3}$

D $\dfrac{3x^2-3-(x-1)(3x-1)+(2x-2)(3x-2)}{\cancel{(x-1)(3x-2)}}=0\cdot D$

$3x^2-3-(3x^2-x-3x+1)+6x^2-4x-6x+4=0$

$\cancel{3x^2}\cancel{-3}\cancel{-3x^2}\cancel{+x}\cancel{+3x}\cancel{-1}+6x^2\cancel{-4x}-6x\cancel{+4}=0$

$+6x^2-6x=0$

$+6x(x-1)=0$

$x=0$

$x=1$ NON ACC

$S=\{0\}$

3- Semplifica le seguenti espressioni razionalizzando i denominatori se necessario : (/10punti)

$$3\sqrt{20}+\frac{3}{2}\sqrt{45}-\sqrt{\frac{18}{25}} \qquad \sqrt{\frac{9}{32}}-\sqrt{\frac{81}{8}}+\frac{\sqrt{3}}{2-\sqrt{2}}$$

• $3\sqrt{20}+\frac{3}{2}\sqrt{45}-\sqrt{\frac{18}{25}}=$

$=6\sqrt{5}+\frac{9}{2}\sqrt{5}-\frac{3}{5}\sqrt{2}=$

$=\frac{21}{2}\sqrt{5}-\frac{3}{5}\sqrt{2}$

• $\sqrt{\frac{9}{32}}-\sqrt{\frac{81}{8}}+\frac{\sqrt{3}}{2-\sqrt{2}}=$

$=\frac{3}{4}\sqrt{\frac{1}{2}}-\frac{9}{2}\sqrt{\frac{1}{2}}+\frac{\sqrt{3}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}=$

$=-\frac{15}{4}\sqrt{\frac{1}{2}}+\frac{2\sqrt{3}+\sqrt{6}}{2}=-\frac{15}{4\sqrt{2}}+\frac{2\sqrt{3}+\sqrt{6}}{2}=-\frac{15}{4\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}+\frac{2\sqrt{3}+\sqrt{6}}{2}=$

$=-\frac{15\sqrt{2}}{8}+\frac{2\sqrt{3}+\sqrt{6}}{2}=\frac{-15\sqrt{2}+8\sqrt{3}+4\sqrt{6}}{8}$

4- Risolvi le seguenti disequazioni, esprimendo le soluzioni nei due modi che conosci: (/30 punti)

$$\frac{1}{2}(4x-2)\le 6x+7 \qquad 4x^2+x\ge 0 \qquad 2x^2-\frac{1}{2}<0$$

$$3-12x^2\le 0 \qquad -x^2+3x-2>0$$

• $\frac{1}{2}(4x-2)\le 6x+7$

$2x-1-6x-7\le 0$

$-4x-8\le 0$

$\frac{\cancel{-4}(x+2)}{\cancel{-4}}\le\frac{0}{-4}$

$x+2\ge 0$

$x\ge -2$

$[-2;+\infty[$

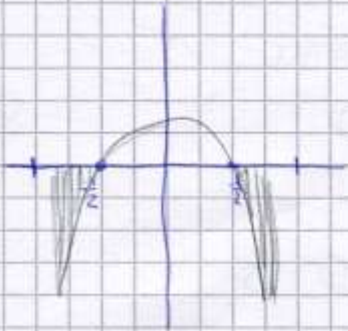
• $3-12x^2 \leq 0$

$\frac{\cancel{3}(1-4x^2)}{\cancel{3}} \leq \frac{0}{3}$

$(1+2x)(1-2x) \leq 0$

$\frac{2x}{2} = \frac{-1}{2} \Rightarrow x = -\frac{1}{2}$

$\frac{-2x}{-2} = \frac{-1}{-2} \Rightarrow x = \frac{1}{2}$



$x \leq -\frac{1}{2} \vee x \geq \frac{1}{2}$

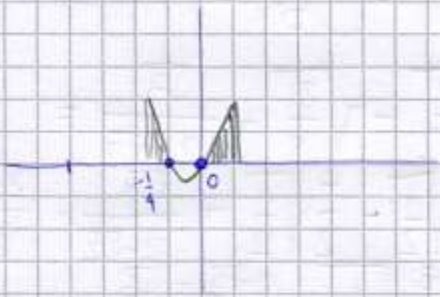
$]-\infty; -\frac{1}{2}] \vee [\frac{1}{2}; +\infty[$

• $4x^2+x \geq 0$

$x(4x+1) \geq 0$

$x=0$

$\frac{4x}{4} = \frac{-1}{4} \Rightarrow x = -\frac{1}{4}$



$x \leq -\frac{1}{4} \vee x \geq 0$

$]-\infty; -\frac{1}{4}] \vee [0; +\infty[$

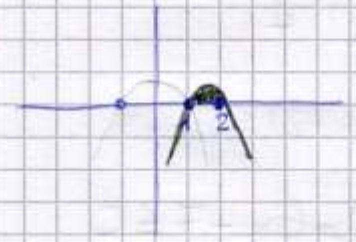
• $-x^2+3x-2 > 0$

$x(-x+1)-2(-x+1) > 0$

$(x-2)(-x+1) > 0$

$x=2$

$x=1$



$1 < x < 2$

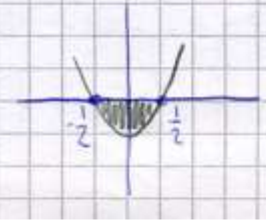
$]1; 2[$

• $2(2x^2-\frac{1}{2}) < 0(2)$

$4x^2-1 < 0$

$(2x+1)(2x-1) < 0$

$x = -\frac{1}{2}$ $x = \frac{1}{2}$



$-\frac{1}{2} < x < \frac{1}{2}$

$]-\frac{1}{2}; \frac{1}{2}[$