1 - Risolvi le seguenti espressioni applicando ove possibile le proprietà delle potenze:

a) $\left(2-\dfrac{1}{2}+\dfrac{1}{5}\right)^{0}+\left(\dfrac{2}{3}+\dfrac{1}{5}\right)^{5}:\left(\dfrac{2}{5}\cdot\dfrac{2}{3}+\dfrac{3}{5}\right)^{3}\cdot\left(\dfrac{13}{5}\right)^{-2}$

svolgimento

$1+\left(\dfrac{10+3}{15}\right)^{5}:\left(\dfrac{4}{15}+\dfrac{3}{5}\right)^{3}\cdot\left(\dfrac{13}{5}\right)^{-2}=1+\left(\dfrac{13}{15}\right)^{5}:\left(\dfrac{13}{15}\right)^{3}\cdot\left(\dfrac{13}{5}\right)^{-2}=1+\left(\dfrac{13}{15}\right)^{2}\cdot\left(\dfrac{13}{5}\right)^{-2}=1+\left(\dfrac{13}{15}\right)^{2}\cdot\left(\dfrac{5}{13}\right)^{2}=$

$=1+\left(\dfrac{13}{15}\cdot\dfrac{5}{13}\right)^{2}=1+\left(\dfrac{1}{3}\right)^{2}=1+\dfrac{1}{9}=\dfrac{10}{9}$

b) $\left((2)^{-2}\right)^{5}:\left(\left(-\dfrac{1}{4}\right)^{6}:\left(-\dfrac{1}{4}\right)^{2}\cdot\left(\dfrac{1}{4}\right)^{-7}\right)^{2}$

svolgimento

$\left(\left(\dfrac{1}{2}\right)^{2}\right)^{5}:\left(\left(-\dfrac{1}{4}\right)^{4}\cdot\left(\dfrac{1}{4}\right)^{-7}\right)^{2}=\left(\dfrac{1}{2}\right)^{10}:\left(\left(\dfrac{1}{4}\right)^{4}\cdot\left(\dfrac{1}{4}\right)^{-7}\right)^{2}=\left(\dfrac{1}{2}\right)^{10}:\left(\left(\dfrac{1}{4}\right)^{-3}\right)^{2}=\left(\dfrac{1}{2}\right)^{10}:\left(\dfrac{1}{4}\right)^{-6}=$

$=\left(\dfrac{1}{2}\right)^{10}:\left(\dfrac{1}{2}\right)^{-12}=\left(\dfrac{1}{2}\right)^{22}=2^{-22}$

2 - Scomponi in fattori i seguenti polinomi:

a) $x^{3}+5x^{2}-4x-20$

svolgimento $\qquad x^{2}(x+5)-4(x+5)=(x+5)(x^{2}-4)=(x+5)(x-2)(x+2)$

b) $5a^{2}+7a+2$

svolgimento $\qquad 5a^{2}+5a+2a+2=5a(a+1)+2(a+1)=(a+1)(5a+2)$

c) $3y^{5}-24y^{2}$

svolgimento $\qquad 3y^{2}(y^{3}-8)=3y^{2}(y-2)(y^{2}+2y+4)$

d) $a^{2}-4a+3$

svolgimento $\qquad a^{2}-3a-a+3=a(a-3)-(a-3)=(a-3)(a-1)$

$b^{5}-16b$

svolgimento $\qquad b(b^{4}-16)=b(b^{2}+4)(b^{2}-4)=b(b^{2}+4)(b+2)(b-2)$

3 - Risolvi le seguenti espressioni:

a) $\left(\dfrac{a}{a^2-4a+4}+\dfrac{a}{a-2}-1\right):\dfrac{a}{2-a}$

svolgimento

$$\left(\dfrac{a}{(a-2)^2}+\dfrac{a}{a-2}-1\right):\dfrac{a}{2-a}=\left(\dfrac{a+a(a-2)-(a-2)^2}{(a-2)^2}\right):\dfrac{a}{2-a}=$$

$$=\left(\dfrac{a+a^2-2a-a^2+4a-4}{(a-2)^2}\right):\dfrac{a}{2-a}=\dfrac{3a-4}{(a-2)^2}:\dfrac{a}{2-a}=\dfrac{3a-4}{(2-a)^2}\cdot\dfrac{2-a}{a}=\dfrac{3a-4}{a(2-a)}$$

b) $\left(\dfrac{b}{b+3}+\dfrac{3}{b}+\dfrac{3}{3+b}\right)^{-1}$

svolgimento

$$\left(\dfrac{b^2+3(b+3)+3b}{b(b+3)}\right)^{-1}=\left(\dfrac{b^2+3b+9+3b}{b(b+3)}\right)^{-1}=\left(\dfrac{b^2+6b+9}{b(b+3)}\right)^{-1}=\left(\dfrac{(b+3)^2}{b(b+3)}\right)^{-1}=\left(\dfrac{b+3}{b}\right)^{-1}=\dfrac{b}{b+3}$$

c) $\dfrac{x^2+2xy+y^2}{x^2-y^2}+\dfrac{3x}{3y-3x}$

svolgimento

$$\dfrac{(x+y)^2}{(x-y)(x+y)}+\dfrac{3x}{-3(-y+x)}=\dfrac{(x+y)^2}{(x-y)(x+y)}-\dfrac{x}{x-y}=\dfrac{(x+y)}{(x-y)}-\dfrac{x}{x-y}=$$

$$=\dfrac{x+y-x}{x-y}=\dfrac{y}{x-y}$$

d) $(x^2-2y)^3-3(2xy-x^3)(2xy+x^3)$

svolgimento

$$x^6-8y^3-6x^4y+12x^2y^2-3(4x^2y^2-x^6)=x^6-8y^3-6x^4y+12x^2y^2-12x^2y^2+3x^6=$$

$$=4x^6-8y^3-6x^4y=4x^6-8y^3-6x^4y=2(2x^6-4y^3-3x^4y)$$