1- Risolvi le seguenti equazioni intere e scrivi le soluzioni reali in ordine crescente. Indica anche se ci sono soluzioni non reali e quante sono:

$17x(3x-1)^2 = 0$ $\quad$ $4x^5 + 4x^3 = 0$ $\quad$ $5x^3 = 6x + 1$

$16x^4 - 25x^2 + 9 = 0$ $\quad$ $17x = (3x+1)(6-x)$

- $17x(3x-1)^2=0$

$17x(3x-1)(3x-1)=0$

$x=0 \vee x=\frac{1}{3}$ doppia $\qquad S=\{0; \frac{1}{3}$ (doppia)$\}$

- $4x^5+4x^3=0$

$4x^3(x^2+1)=0$

$x=0$ tripla $\vee x^2=-1$ $\qquad S=\{0$ (tripla)$\}$ e due soluzioni $\notin \mathbb{R}$

$\Downarrow$

$x_{1,2} \notin \mathbb{R}$

- $5x^3=6x+1$ $\qquad$ Ruffini $\qquad a=-1$

$5x^3-6x-1=0$

$(x+1)(5x^2-5x-1)=0$

$x=-1 \vee 5x^2-5x-1=0$

$\Delta=25+20=45$

	5	0	-6	-1
-1		-5	5	1
	5	-5	-1	0

$(x+1)(5x^2-5x-1)=0$

$x_{1,2}=\frac{5\pm\sqrt{45}}{10}=\frac{5\pm 3\sqrt{5}}{10}$

$x_1=\frac{5+3\sqrt{5}}{10}$

$x_2=\frac{5-3\sqrt{5}}{10}$

$S=\{-1; \frac{5-3\sqrt{5}}{10}; \frac{5+3\sqrt{5}}{10}\}$

$16x^4-25x^2+9=0$ $\qquad x^2=t$

$16t^2-25t+9=0$

$16t^2-16t-9t+9=0$

$16t(t-1)-9(t-1)=0$

$(t-1)(16t-9)=0$

$t=1 \vee t=\frac{9}{16}$

$t_1=1 \Rightarrow x^2=1 \Rightarrow x=\pm 1$

$t_2=\frac{9}{16} \Rightarrow x^2=\frac{9}{16} \Rightarrow x=\pm\frac{\sqrt{9}}{\sqrt{16}} \Rightarrow x=\pm\frac{3}{4}$

$S=\{-1; -\frac{3}{4}; \frac{3}{4}; 1\}$

$17x=(3x+1)(6-x)$

$17x=18x-3x^2+6-x$

$18x-3x^2+6-x-17x=0$

$-3x^2+6=0$

$-3(x^2-2)=0$

$x=-\sqrt{2} \vee x=+\sqrt{2}$ $\qquad S=\{-\sqrt{2}; \sqrt{2}\}$

2- Risolvi le seguenti equazioni fratte:

$$\frac{2}{6+2x}+\frac{x^2-x}{x^2+6x+9}=0 \qquad\qquad \frac{3x^2+x}{3x^2-5x-2}=\frac{x}{2-x}$$

- $\frac{2}{6+2x}+\frac{x^2-x}{x^2+6x+9}=0$

$\frac{2}{2(3+x)}+\frac{x^2-x}{(x+3)^2}=0$ CE. $x\neq -3$

$\frac{2(x+3)+2(x^2-x)}{2(x+3)^2}=0 \cdot D$

$\cancel{2x}+6+2x^2-\cancel{2x}=0$

$2x^2+6=0$

$2(x^2+3)=0$

$x^2=-3$ impossibile

$\Downarrow$

$x_{1,2}\notin \mathbb{R}$

$S=$ Due soluzioni $\notin \mathbb{R}$ quindi $S=\emptyset$

- $\frac{3x^2+x}{3x^2-5x-2}=\frac{x}{2-x}$

$3x^2-5x-2$

$\Delta=25-4(3)(-2)=25+24=49$

$x_{1,2}=\frac{5\pm 7}{6} = \frac{12}{6}=2 \quad ; \quad -\frac{2}{6}=-\frac{1}{3}$

$3(x+\frac{1}{3})(x-2)=(3x+1)(x-2)$

$\frac{3x^2+x}{(3x+1)(x-2)}-\frac{x}{2-x}=0$

$\frac{3x^2+x}{(3x+1)(x-2)}+\frac{x}{x-2}=0$ CE. $x\neq -\frac{1}{3} \land x\neq 2$

$\frac{3x^2+x+3x^2+x}{(3x+1)(x-2)}=0 \cdot D$

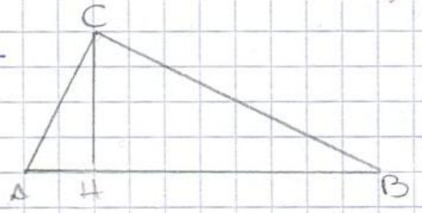
$6x^2+2x=0$

$2x(3x+1)=0$

$x=0 \lor x=-\frac{1}{3}$ non accettabile $S=\{0\}$

3- In un triangolo rettangolo l’altezza relativa all’ipotenusa è i $\frac{4}{3}$ della proiezione di uno dei cateti sull’ipotenusa e la misura dell’ipotenusa è 25 cm. Determina il perimetro e l’area del triangolo.

3-



$\overline{CH} = \frac{4}{3}\overline{AH}$

$\overline{AB} = 25\,cm$

$P = ?$

$A = ?$

$\overline{AH} = x$

$\overline{CH}^2 = \overline{AH} \cdot \overline{HB}$

$\left(\frac{4}{3}x\right)^2 = x(25-x)$

$\frac{16}{9}x^2 = 25x - x^2$

$\frac{16}{9}x^2 + x^2 - 25x = 0$

$\frac{16x^2 + 9x^2 - 225x}{9} = 0$

$25x^2 - 225x = 0$

$25x(x-9) = 0 \qquad x = 0 \vee x = 9$ (per $x=0$ il triangolo si riduce a un punto)

$\overline{CH} = \frac{4}{3} \cdot 9 = 12\,cm \qquad \overline{HB} = \overline{AB} - \overline{AH} = 25\,cm - 9\,cm = 16\,cm$

$\overline{AC}^2 = \overline{AB} \cdot \overline{AH}$

$\overline{AC} = \sqrt{25\,cm \cdot 9\,cm} = \sqrt{225\,cm^2} = 15\,cm$

$\overline{BC}^2 = \overline{AB} \cdot \overline{HB}$

$\overline{BC} = \sqrt{25\,cm \cdot 16\,cm} = \sqrt{400\,cm^2} = 20\,cm$

$P = \overline{AB} + \overline{AC} + \overline{BC} = (25+15+20)\,cm = 60\,cm$

$A = \frac{\overline{AC} \cdot \overline{BC}}{2} = \frac{15\,cm \cdot 20\,cm}{2} = 150\,cm^2$

oppure

$A = \frac{\overline{CH} \cdot \overline{AB}}{2} = \frac{12\,cm \cdot 25\,cm}{2} = 150\,cm^2$